International Journal of Frontline Research in
Engineering and Technology
Journal homepage: https://frontlinejournals.com/ijfret/
ISSN: 2945-4840 (Online)

(Research Article)

Scrutiny of principle of energy survival in beam-columns and analytical solution of governing partial nonlinear differential equation by new approach AYM

MR Akbari [1, 2, *], Sara Akbari [2] and Esmaeil Kalantari [2]

[1] *Department of Civil Engineering and Chemical Engineering, University of Tehran, Tehran, Iran.*
[2] *Department of Chemical Engineering, Faculty of Chemical Engineering, Islamic Azad University, Ghaemshahr, Iran.*

International Journal of Frontline Research in Engineering and Technology, 2022, 01(01), 034–041

Publication history: Received on 17 February 2022; revised on 29 March 20222; accepted on 31 March 2022

Article DOI: https://doi.org/10.56355/ijfret.2022.1.1.0002

Abstract

In this manuscript, we investigate and solve a complicated highly nonlinear differential equations in the beam-column in the Energy Survival Principal equation on the beam-column to show the ability of the first invented method which we named it AYM (Akbari Yasna's Method). Certainly, we know that nonlinear beam-column process are very complex, and the governing differential equations governing them are nonlinearity complex. In this paper we present a new analytical solution which can easily analyze all such problems and make a great evolution in the nonlinear designing in the solid engineering (civil and mechanical and etc.). Finally, this scientific approach can create a great phenomenon in the analytical solution of nonlinear problems in engineering sciences, especially in the solids engineering.

Keywords: New Approach; Akbari-Yasa's-Method (AYM); Mode's frequencies and Critical Load; Nonlinear Differential Equation; Principle of Energy Survival

1. Introduction

In the study, our aims introduce of accuracy, capabilities and power for solving complex non-linear differential in the solids engineering on the beam-column. AYM method can be successfully applied in various engineering fields such as mechanics (solid and fluid), electronics, petroleum industry, industrial design [1,2], and also in applied sciences (physics), economics and so on. It is worth noting that these methods is convergent at any form of differential equations, including any number of initial and boundary conditions. During the solution procedure, it is not required to convert or simplify the exponential, trigonometric and logarithmic terms, which enables the user to obtain a highly precise solution. Besides, the methodology behind these techniques are completely understandable, easy to use, and users with common knowledge of mathematics will be capable of solving the most complicated equations at low calculation cost.As all experts know most of engineering actual systems behavior in practical are nonlinear process and analytical scrutiny these nonlinear problems are difficult or sometimes impossible. Our purpose is to enhance the ability of solving the mentioned nonlinear differential equations at chemical engineering and similar issues with a simple and innovative approach which entitled "Akbari-Yans's Method" or "AYM". He's Amplitude Frequency Formulation method [3-5] which was first presented by Ji-Huan He gives convergent successive approximations of the exact solution and Homotopy perturbation technique HPM [6]. It is necessary to mention that the above methods do not have this ability to gain the solution of the presented problem in high precision and accuracy so nonlinear differential equations such as the presented problem in this case study should be solved by utilizing new approaches like AGM [7-13] that created by Mohammadreza Akbari(in 2014).In recent years, analytical methods in solving nonlinear differential equations have been presented and created by Mohammadreza Akbari, these methods are called AYM (Akbari Yasna's Method in April 2020) and ASM[14] (Akbari Sara's Method in August 2019) and AKLM (Akbari Kalantari Leila Method in August

* Corresponding author: MR Akbari
Department of Civil Engineering and Chemical Engineering,University of Tehran, Tehran, Iran.

2020).These example somehow can be considered as complicated cases to deal with for all of the existed analytical methods especially in the design slides engineering , which means old methods cannot resolve them precisely or even solve them in a real domain.

2. Mathematical formulation of the problem

We consider the beam-columns which boundary conditions are different as follows:

2.1 Beam-column in positions of the Hinged-Hinged

Governing partial nonlinear differential equation on the beam-columns (according to the principle of energy survival) is as follows:

$$\frac{1}{2}\int_0^x EI\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2 dx - \frac{1}{2}\int_0^x m\left(\frac{\partial v(x,t)}{\partial t}\right)^2 dx - \frac{p}{2}\int_0^x \left(\frac{\partial v(x,t)}{\partial x}\right)^2 dx = 0 \quad (1)$$

We consider a beam-column as follows:

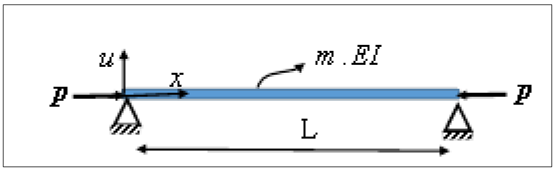

Figure 1 Schematic of the problem for beam-column

The shapes of mode at position of the Hinged-Hinged as follows:

$$\text{Hinged, Hinged} \rightarrow \quad \phi x := C\sin\left(\frac{n\,\pi\cdot x}{L}\right) \quad (2)$$

The answer of partial nonlinear differential equations Eqs.(1) by *AYM* method (*A*kbari *Y*asna's *M*ethod) is obtained as follows:

$$u(x,t) = C\sin\left(\frac{n\,\pi\,x}{L}\right) e^{\frac{i\,n\,\pi\sqrt{-\pi^2 n^2 EI + L^2 p}}{\sqrt{m}\,L^2}t} \quad (3)$$

$$C = \frac{2uo\left(1-(-1)^n\right)}{\pi\,n}$$

Or as:

$$u(x,t)=\frac{2\,uo\,(1-(-1)^n)}{n\,\pi}\sin\left(\frac{\pi\,x}{L}\right)\left[\sin\left(\frac{n\,\pi\sqrt{L^2\,p-E\,I\,\pi^2}\;t}{\sqrt{m}\;L^2}\right)+i\cos\left(\frac{n\,\pi\sqrt{L^2\,p-E\,I\,\pi^2}\;t}{\sqrt{m}\;L^2}\right)\right] \quad (3a)$$

The parameter (***i***) in the Eqs.(3,3a) is an imaginary value.

Modes frequency and critical load are obtained as follows:

$$\omega=\frac{n\,\pi}{L^2}\sqrt{\frac{E\,Ie\,\pi^2\,n^2-L^2\,p}{m}}\;;\quad Pcr:=\frac{\pi^2 E\,I}{L^2} \quad (4)$$

By selecting the physical values as below:

$$E:=2500;\,I:=0.0002;\,m:=40;\,L:=1;\,uo:=0.01;\,n:=1\,;\,p:=\frac{1}{3}\,Pcr \quad (5)$$

Comparing the achieved solutions by Numerical Method and ***AYM*** method as:

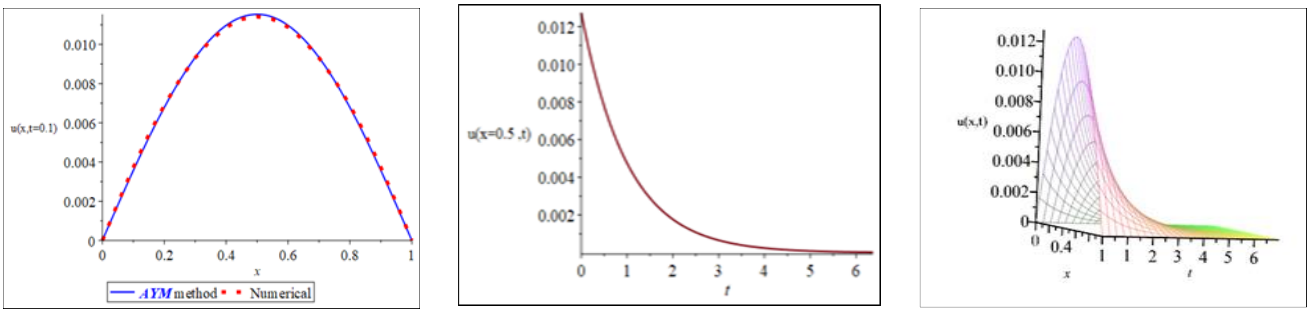

Figure 2 A comparison between AYM and Numerical solution

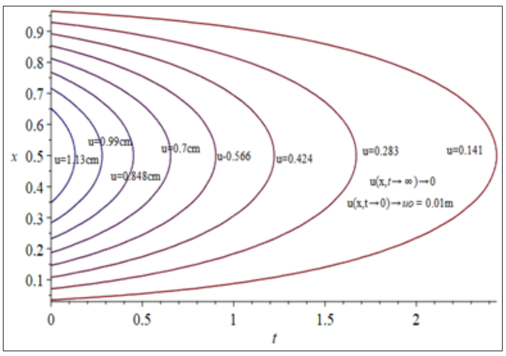

Figure 3 Charts of contour of AYM solution for u(x,t)

As we can see from the contour diagrams,at zero time(***t=0***) the value of ***u*** equals (***uo=0.01***) and at infinite times ($t\to\infty$)the value of u tends to zero ($u\to uo=0.01m$).

2.2 Beam-column in positions of the Fixed-Fixed

As in the previous section, governing partial nonlinear differential equation on the beam-columns (according to the principle of energy survival) is as follows:

$$\frac{1}{2}\int_0^x EI\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2 dx - \frac{1}{2}\int_0^x m\left(\frac{\partial v(x,t)}{\partial t}\right)^2 dx - \frac{p}{2}\int_0^x \left(\frac{\partial v(x,t)}{\partial x}\right)^2 dx = 0 \tag{6}$$

We consider a beam-column as follows:

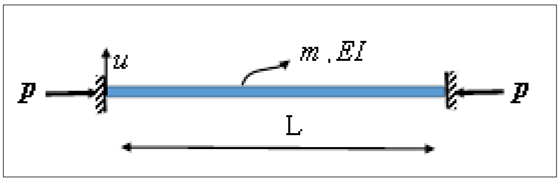

Figure 4 Schematic of the problem for beam-column. column

The shapes of mode at position of the Fixed-Fixed as follows:

$$\text{Fixeded, Fixeded} \rightarrow \phi x := C\left(1 - \cos\left(\frac{2\pi x}{L}\right)\right) \tag{7}$$

The answer of partial nonlinear differential equations Eqs.(6) by *AYM* method (*A*kbari *Y*asna's *M*ethod) is obtained as follows:

$$u(x,t) = C\left(1 - \cos\left(\frac{2 n \pi x}{L}\right)\right) e^{\frac{2 i \pi n \sqrt{3}\sqrt{-4\pi^2 n^2 E I + L^2 p}\; t}{3\sqrt{m}\, L^2}} \tag{8}$$

$$C = \frac{2\, uo}{3}$$

Or as:

$$u = \frac{2\, uo}{3}\left(1 - \cos\left(\frac{2 n \pi x}{L}\right)\right)\left[\sin\left(\frac{2\sqrt{3}\, n \pi \sqrt{L^2 p - 4 n^2 E I \pi^2}\; t}{3\sqrt{m}\, L^2}\right) + i\cos\left(\frac{2\sqrt{3}\, n \pi \sqrt{L^2 p - 4 n^2 E I \pi^2}\; t}{3\sqrt{m}\, L^2}\right)\right] \tag{8a}$$

The parameter (***i***) in the Eqs.(8,8a) is an imaginary value.

Modes frequency and critical load are obtained as follows:

$$\omega := \frac{2 n \pi \sqrt{3}}{3 L^2}\sqrt{\frac{4 E I \pi^2 n^2 - L^2 p}{m}} \quad ; \quad Pcr := \frac{4\pi^2 E I}{L^2} \tag{9}$$

By selecting the physical values as below:

$$E := 2500; I := 0.0002; m := 40; L := 1; uo := 0.01; n := 1; p := \frac{1}{3} Pcr \quad (10)$$

The achieved solutions by ***AYM*** method as follows:

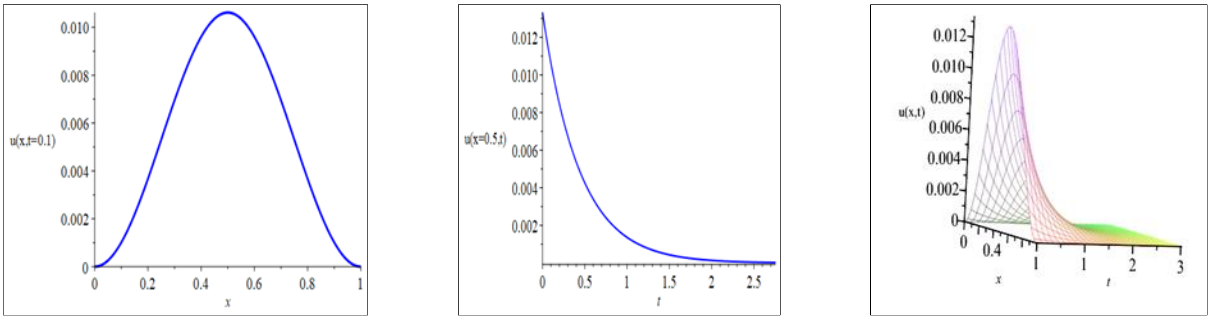

Figure 5 A comparison between *AYM* and Numerical solution

2.3 Beam-column in positions of the Fixed-Fixed in the case where the cross section of the beam-column is variable

Governing partial nonlinear differential equation on the beam-columns (according to the principle of energy survival) for the case where the cross-sectional area and material along the beam of the column are different as follows:

$$\frac{1}{2}\int_0^x EI\left(\frac{\partial^2 v(x,t)}{\partial x^2}\right)^2 dx - \frac{1}{2}\int_0^x m\left(\frac{\partial v(x,t)}{\partial t}\right)^2 dx - \frac{p}{2}\int_0^x \left(\frac{\partial v(x,t)}{\partial x}\right)^2 dx = 0 \quad (11)$$

$$I := Io(1 - \alpha x)^4 ; E := Eo(1 + \beta x) ; m := mo(1 + \varepsilon x)^2 \quad (12)$$

We consider a beam-column as follows:

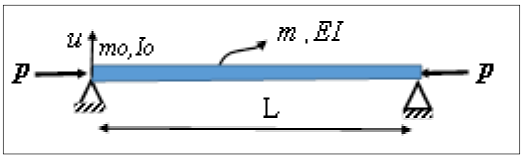

Figure 6 Schematic of the problem for beam-column

The shapes of mode at position of the Hinged-Hinged as follows:

$$\textit{Hinged, Hinged} \rightarrow \quad \phi x := C \sin\left(\frac{n\pi \cdot x}{L}\right) \quad (13)$$

The answer of partial nonlinear differential equations Eqs.(11) by *AYM* method (*A*kbari *Y*asna's *M*ethod) is obtained as follows:

$$u(x,t) = C \sin\left(\frac{\pi x}{L}\right) e^{\frac{3 i \sqrt{2 \psi\, mo\, \Delta}\, t}{2\, mo\, L^2 \Delta}} \qquad C := \frac{4\, uo}{\pi} \tag{14}$$

Or as:

$$u(x,t) = \frac{4\, uo}{\pi} \sin\left(\frac{\pi x}{L}\right) \left[\sin\left(\frac{3 \sqrt{2 \psi\, mo\, \Delta}\; t}{2\, mo\, L^2 \Delta}\right) + i \cos\left(\frac{3 \sqrt{2 \psi\, mo\, \Delta}\; t}{2\, mo\, L^2 \Delta}\right)\right] \tag{14a}$$

The parameter (***i***) in the Eqs.(14,14a) is an imaginary value.And parameters are:

$$\begin{aligned}\psi = {} & Eo\, Io\, \beta\, \alpha^4 L^5 \left(\pi^6 - \frac{125}{6}\pi^4 + \frac{3605}{18}\pi^2 - \frac{58240}{81}\right) \\ & - \frac{24}{5}\pi^2 \alpha^3 L^4 Eo\, Io \left(\pi^4 - \frac{125}{9}\pi^2 + \frac{3605}{54}\right)\left(\beta - \frac{\alpha}{4}\right) \\ & + 9\pi^2 EoIo\, \alpha^2 L^3 \left(\beta - \frac{2\alpha}{3}\right)\left(\pi^4 - \frac{25}{3}\pi^2 + \frac{640}{27}\right) \\ & - 6\pi^4 L^2 \left[\frac{4}{3}\alpha \pi^2 Eo\, Io \left(\beta - \frac{3\alpha}{2}\right) - \frac{50}{9}\alpha\, Eo\, Io \left(\beta - \frac{3\alpha}{2}\right) \right. \\ & \left. + p\right] + 3\pi^4 L\, Eo\, Io (\beta - 4\alpha)\left(\pi^2 - \frac{16}{9}\right) + 6\, Eo\, Io\, \pi^6\end{aligned} \tag{15}$$

$$\Delta = \varepsilon^2 L^2 \left(\pi^2 - \frac{25}{6}\right) + \varepsilon L \left(3\pi^2 - \frac{16}{3}\right) + 3\pi^2$$

Modes frequency and critical load are obtained as follows:

$$\omega = \frac{\sqrt{2 \psi\, mo\, \Delta}}{2\, mo\, L^2 \Delta} \tag{16}$$

$$\begin{aligned}Pcr = {} & \frac{Eo\, Io}{4860\, \pi^4 L^2} \{810 L^5 \pi^6 \alpha^4 \beta - 16875 L^5 \pi^4 \alpha^4 \beta + 972 L^4 \pi^6 \alpha^4 \\ & - 3888 L^4 \pi^6 \alpha^3 \beta + 162225 L^5 \pi^2 \alpha^4 \beta - 13500 L^4 \pi^4 \alpha^4 \\ & + 54000 L^4 \pi^4 \alpha^3 \beta - 4860 L^3 \pi^6 \alpha^3 + 7290 L^3 \pi^6 \alpha^2 \beta \\ & - 582400 L^5 \alpha^4 \beta + 64890 L^4 \pi^2 \alpha^4 - 259560 L^4 \pi^2 \alpha^3 \beta \\ & + 40500 L^3 \pi^4 \alpha^3 - 60750 L^3 \pi^4 \alpha^2 \beta + 9720 L^2 \pi^6 \alpha^2 \\ & - 6480 L^2 \pi^6 \alpha \beta - 115200 L^3 \pi^2 \alpha^3 + 172800 L^3 \pi^2 \alpha^2 \beta \\ & - 40500 L^2 \pi^4 \alpha^2 + 27000 L^2 \pi^4 \alpha \beta - 9720 L \pi^6 \alpha + 2430 L \pi^6 \beta \\ & + 17280 L \pi^4 \alpha - 4320 L \pi^4 \beta + 4860 \pi^6\}\end{aligned} \tag{17}$$

By selecting the physical values as below:

$$\begin{aligned}& Eo := 2400;\ Io := 0.001;\ mo := 25;\ L := 2;\ uo := 0.01 \\ & \alpha := 0.1;\ \beta := 0.01;\ \varepsilon := 0.01;\ p := \frac{Pcr}{3}\end{aligned} \tag{18}$$

Comparing the achieved solutions by Numerical Method and *AYM* method as follows:

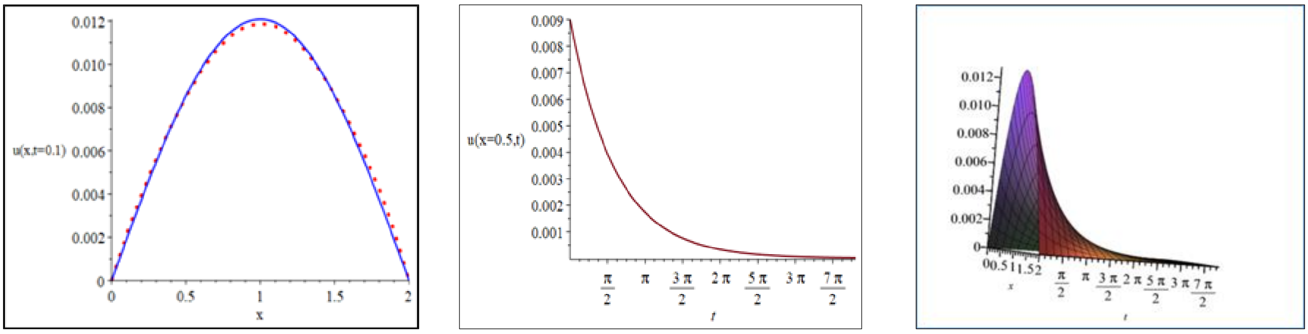

Figure 7 A comparison between *AYM* and Numerical solution

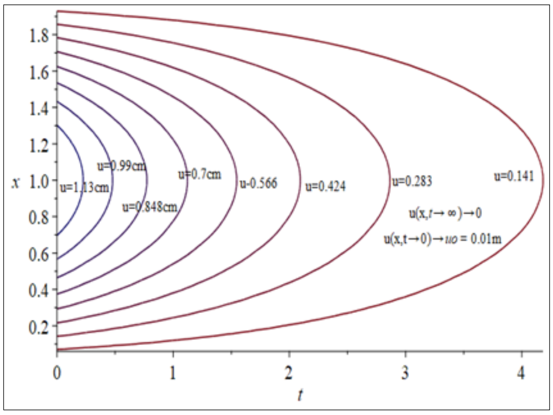

Figure 8 Charts of contour of AYM solution for u(x,t)

As we can see from the contour diagrams,at zero time(***t=0***) the value of ***u*** equals (***uo=0.01***) and at infinite times ($t \to \infty$)the value of u tends to zero $(u \to uo = 0.01m)$.

3. Conclusion

In this article, we proved that with this new method, all kinds of complex practical problems related to nonlinear differential equations in the design of beam-column can be easily solved analytically. Obviously, most of the phenomena of solid engineering are nonlinear, so it is quite difficult to study and analyze nonlinear mathematical equations in this area, also we wanted to demonstrate the strength, capability and flexibility of the new method AYM (Akbari-Yasna's Method).This method is newly created and it can have high power in analytical solution of all kinds of industrial and practical problems in engineering fields and basic sciences for complex nonlinear differential equations.

Compliance with ethical standards

Disclosure of conflict of interest

There is no conflict of interest.

References

[1] Bruce A. Finlayson, Brigette M. Rosendall. University of Washington Seattle,WA 98195-1750 ,Bechtel Technology and Consulting San Francisco, CA. 94119-3965.

[2] D Zwillinger. Handbook of Differential Equations. 1992.

[3] JH He. An improved amplitude-frequency formulation for nonlinear oscillators. International Journal of Nonlinear Sciences and Numerical Simulation. 2008; 9(2): 211.

[4] ZF Ren, GQ Liu, YX Kang, HY Fan, HM Li, XD Ren, WK Gui. Application of he's amplitude frequency formulation to nonlinear oscillators with discontinuities, Physica Scripta. 2009; 80: 45003.

[5] JH He. Approximate analytical solution of Blasiu's equation, Commun. Nonlin. Sci. Numer. Simul. 1998; 3: 260–263.

[6] JH He. Homotopy perturbation technique, J. Comput.Methods Appl. Mech. Engrg. 1999; 17(8): 257–262.

[7] Book Nonlinear Dynamic in Engineering by Akbari-Ganji's MethodISBN-13: 978-1514401699, ISBN-10: 151440169X.

[8] MR Akbari, Sara Akbari, Esmaeil Kalantari. 'Akbari-Ganji‟s method "AGM" to chemical reactor design for non-isothermal and non-adiabatic of mixed flow reactors'Journal of Chemical Engineering and Materials Science. 2020; 11(1): 1-9.

[9] MR Akbari, DD Ganji, M Nimafar. Significant progress in solution of nonlinear equations at displacement of structure and heat transfer extended surface by new AGM approach, Frontiers of Mechanical Engineering Journal. 2014.

[10] AK Rostami, MR Akbari DD, Ganji S. Heydari, Investigating Jeffery-Hamel flow with high magnetic field and nanoparticle by HPM and AGM, Cent. Eur. J. Eng. 2014; 4(4): 357-370.

[11] MR Akbari, DD Ganji, A Majidian, AR Ahmadi. Solving nonlinear differential equations of Vanderpol, Rayleigh and Duffing by AGM, Frontiers of Mechanical Engineering. April 2014.

[12] DD Ganji, MR Akbari, AR Goltabar. Dynamic Vibration Analysis for Non-linear Partial Differential Equation of the Beam - columns with Shear Deformation and Rotary Inertia by AGM, Development and Applications of Oceanic Engineering (DAOE). 2014.

[13] MR Akbari, DD Ganji, AR Ahmadi, Sayyid H, Hashemi kachapi. Analyzing the Nonlinear Vibrational wave differential equation for the simplified model of Tower Cranes by (AGM), Frontiers of Mechanical Engineering March 2014; 9(1): 58-70.

[14] MR Akbari, M Nimafar DD Ganji MM. Akbarzade. Scrutiny of non-linear differential equations Euler Bernoulli beam with large rotational deviation by AGM Springer. December 2014.

[15] MR Akbari, Sara Akbari, Esmaeil Kalantari. A Study about Exothermic Chemical Reactor by ASM Approach Strategy Crimson Publishers Wings to the Research,Published: 17 March 2020.